

**AMCS243/CS243/EE243**  
**Probability and Statistics**  
**Spring 2014**

**Final/Qualifying Exam:**  
**Monday May 19, 9:00am-11:50am**

**VERSION A**

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**ID:**

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**Problem 1: (15pts)** In a class, 40% of the students are from Saudi Arabia, 25% from China, the remaining are from elsewhere. In addition, among the Saudi students, 55% are male, among the Chinese students, 45% are male, and among the students from elsewhere, 50% are male.

a) What is the probability that a random student from this class is a Saudi female?

b) What is the probability that a random student from this class is female?

c) If a random student happens to be male, what is the probability that he is Chinese?

**Problem 2: (15pts)** The number of students registered for a new course in Statistics follows a Poisson distribution with mean 100. The instructor of this course has decided to split the class into 2 sections if the number of students registered is at least 120, otherwise she will have only 1 section.

- a) Give an expression for the exact probability that this instructor has 2 sections for her course.

- b) Compute a numerical approximation to the probability under a).

**Problem 3: (15pts)** Circle the correct answer to each of the following questions:

1. If  $X$  and  $Y$  are two events such that  $P(X)=0.3$ ,  $P(Y)=0.5$  and  $P(X \cup Y)=0.8$  then  $X$  and  $Y$  are:
  - A. Mutually exclusive, but not necessarily independent
  - B. Independent and mutually exclusive
  - C. Independent but not mutually exclusive
  - D. Dependent and mutually exclusive
  - E. Not necessarily independent and not necessarily mutually exclusive
  
2. Which one of the following is not a statistic?:
  - A. The median of the observations
  - B. The mean of the data
  - C. The sample variance
  - D. The population mean
  - E. The maximum of the observations
  
3. Assume  $X$  and  $Y$  are two independent standard normal random variables. What is the variance of  $X^2 + Y^2$ ?:
  - A. 0
  - B. 1
  - C. 2
  - D. 4
  - E. We need to know the joint distribution of  $X$  and  $Y$  to answer this question
  
4. To decrease the length of a confidence interval, we can:
  - A. Increase the confidence level
  - B. Increase the variance
  - C. Increase the mean
  - D. Increase the sample size
  - E. None of the above
  
5. Consider the pdf defined by  $f(x)=3x^2$  for  $0 < x < 1$  and  $f(x)=0$  otherwise. What is the probability  $P(0 < X < 0.5)$ ?:
  - A. 0
  - B.  $1/2$
  - C.  $1/4$
  - D.  $1/8$
  - E. None of the above

**Problem 4: (15pts)** Let  $X_1$  and  $X_2$  be two independent random variables, each with a Poisson distribution with parameter  $\lambda_1$  and  $\lambda_2$ , respectively. Compute the conditional distribution of  $X_1$  given  $X_1+X_2=n$ , where  $n$  is an integer. If this conditional distribution has a specific name, then mention it and give its parameter(s).

**Problem 5: (15pts)** Let  $X_1, \dots, X_n$  be a random sample from a uniform  $U(-\theta, 2\theta)$  distribution,  $\theta > 0$ .

a) Derive a method-of-moment estimator  $\hat{\theta}_{\text{MM}}$  of  $\theta$ .

b) Derive the probability density function  $f(x)$  and the cumulative distribution function  $F(x)$  for the uniform  $U(-\theta, 2\theta)$  distribution. Make sure to describe these two functions for any real value  $x$ .

c) Derive the maximum likelihood estimator  $\hat{\theta}_{\text{ML}}$  of  $\theta$ .

**Problem 6: (15pts)** A random sample of size  $n = 30$  is taken from a normal  $N(\mu, \sigma = 3)$  distribution. If the sum of the  $n$  observations is 56, find:

a) The p-value of the test of  $H_0: \mu = 1.8$  against the alternative  $H_1: \mu \neq 1.8$

b) The probability of a type II error at  $\mu = 3$  when the level of significance is  $\alpha = 5\%$ .



**Problem 7: (10pts)** The number of items produced by a factory within a week is a random variable with mean 50.

(a) What is the highest probability that the production of the following week is at least 75 items?

(b) If in addition the variance of that random variable is 25, what is the lowest probability that the production of the following week is between 40 and 60 items?