AMCS243/CS243/EE243

Probability and Statistics

Fall 2013

Final Exam:

Sunday Dec. 8, 3:00pm-5:50pm

VERSION A

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ID:

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Problem 1: (15pts) A manufacturing firm employs three plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The probability of a defective product from plan 1 is 0.01, the probability of a defective product from plan 2 is 0.03, and the probability of a defective product from plan 3 is 0.02. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

Problem 2: (15pts) A study was conducted at KAUST to estimate the difference in the amounts of a chemical measured at two different independent locations, A and B, on the Red Sea. Although 15 observations were collected at location A, only 12 observations were collected at location B. The 15 observations from location A had a sample average of 3.84 and a sample standard deviation of 3.07. The 12 observations from location B summed to 17.88 and had a sample variance of 0.64. Compute a 95% confidence interval for the difference in the true mean chemical at these two locations assuming that the observations came from two independent normal populations with different variances.

Problem 3: (15pts) Circle the correct answer to each of the following questions:

1. If X and Y are two independent random variables, each with uniform *U(0,2)* distribution, what is the variance of $X+2Y$?:
   A. 1/3
   B. 2/3
   C. 4/3
   D. 5/3
   E. None of the above

2. If X and Y are two independent random variables, each with uniform *U(0,2)* distribution, what is the probability of $X \geq 1 + Y$?:
   A. 1/4
   B. 1/8
   C. 1/2
   D. 7/8
   E. 3/4

3. Assume X and Y are two independent standard normal random variables. What is the mean of $X^2 + Y^2$?:
   A. 0
   B. 1
   C. 2
   D. 4
   E. We need to know the joint distribution of X and Y to answer this question

4. Let the joint probability density of X and Y be $f(x,y) = \frac{6}{\pi} e^{-9x^2-4y^2}$ for $(x,y) \in \mathbb{R}^2$. What is the marginal probability density of Y?
   A. $\frac{3}{\sqrt{\pi}} e^{-9x^2}$
   B. $\frac{6}{\sqrt{\pi}} e^{-9x^2}$
   C. $\frac{\sqrt{2}}{\sqrt{\pi}} e^{-4y^2}$
   D. $\frac{2}{\sqrt{\pi}} e^{-4y^2}$
   E. None of the above

5. In a hypothesis test, if we want to increase the power of the test, we can:
   A. Decrease the probability of type I error allowed
   B. Increase the probability of type II error allowed
   C. Increase the probability of type I error allowed
   D. Decrease the sample size
   E. None of the above

Problem 4: (15pts) Let X and Y be independent Binomial random variables with identical parameters n and p. Compute the conditional distribution of X given $X+Y=m$. If this conditional distribution has a specific name, then mention it and give its parameter(s).

Problem 5: (15pts) Let $X_1,\ldots,X_n$ be a random sample from a uniform $U(-\theta,\theta)$ distribution, $\theta>0$.

a) Derive a method-of-moment estimator $\hat{\theta}_{\mathrm{MM}}$ of θ.

b) Derive the probability density function $f(x)$ and the cumulative distribution function $F(x)$ for the uniform $U(-\theta,\theta)$ distribution. Make sure to describe these two functions for any real value x.

c) Derive the maximum likelihood estimator $\hat{\theta}_{\mathrm{ML}}$ of θ.

d) Show that $\hat{\theta}_{\mathrm{ML}}$ is a biased estimator of θ and propose an unbiased estimator $\hat{\theta}_{\mathrm{UB}}$ of θ based on $\hat{\theta}_{\mathrm{ML}}$.

e) Compute the mean squared error (MSE) of $\hat{\theta}_{\mathrm{UB}}$.

Problem 6: (15pts) A random sample of size $n = 30$ is taken from a normal $N(\mu, \sigma = 3)$ distribution. If the sum of the n observations is 56, find:

a) The p-value of the test of $H_0: \mu = 1.8$ against the alternative $H_1: \mu \neq 1.8$

b) The probability of a type II error at $\mu = 4$ when the level of significance is $\alpha = 5\%$.

Problem 7: (10pts)

(a) Let X be a random variable with mean 0 and finite variance σ^2. Use Markov's Theorem to show that for any $a>0$:

$$P(X \geq a) \leq \frac{\sigma^2}{\sigma^2 + a^2}$$

(b) Generalize the inequality in (a) to the case of a random variable X with mean μ and finite variance σ^2.