MCSE Core Qualifying Exam: PDEs **19 May 2012**

For full credit, answer all questions in both parts #1 – #2 (both sides of the page). Attach extra work as needed and copy your final answers into the blanks on this page.

1) Consider the differential equation for $u(x, y)$: $\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = 1$.

(a) Classify this equation as to linearity (or not), order, and type (elliptic, hyperbolic, parabolic).

(b) Solve the equation subject to $u(0, y) = y$.

2) Consider the differential equation $\nabla^2 u = 0$ in the bounded sector defined by $0 < r < a$ and $0 < \theta < \alpha$, where (r, θ) are polar coordinates (so $\nabla^2 u \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$).

(a) Classify this equation as to linearity (or not), order, and type (elliptic, hyperbolic, parabolic).

(b) Solve the equation subject to $u(r, 0) = u(r, \alpha) = 0$ and $u(a, \theta) = f(\theta)$, invoking boundedness of the solution as $r \to 0$, but do not bother to evaluate any resulting integral(s).

(c) Now assume that $f(\theta) = \sin(17\ \pi\theta/\alpha)$ and find $u(r, \theta)$. In this part, be sure to evaluate all integrals.